$$\therefore \cos\varphi = \frac{ab}{r\sqrt{(a^2+b^2-r^2)}},\ \sin\varphi = \frac{\sqrt{[r^2(a^2+b^2-r^3)-a^2b^2]}}{r\sqrt{(a^2+b^2-r^2)}}.$$

$$\therefore\ n = \frac{\sqrt{[r^2(a^2+b^2-r^2)-a^2b^2]}}{r\sqrt{(a^2-r^2)}} = \frac{\sqrt{(r^2-b^2)}}{r}.$$

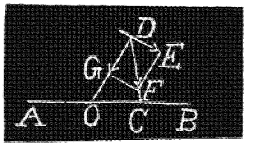
$$\therefore\ r^2-b^2=r^2n^2, \text{ or } r=\frac{b}{\sqrt{(1-n^2)}}.$$

129. Proposed by **B. F. FINKEL, A. M., M. Sc.**, Professor of Mathematics and Physics in Drury College, Springfield, Mo.

Two spheres whose masses are M_1 and M_2 are a units apart, and attract each other with a force $=M_1M_2/a_2$. Find work done in carrying a unit mass from the center point between them a distance r in a direction θ with line of centers.

Solution by **G. B. M. ZERR, A. M., Ph. D.**, Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

Let $B=m_1$, $A=m_2$, $m_1>m_2$; C, the centroid of m_1, m_2; $OC=c$, $OD=x$, $DC=y$, $\angle DOC=\theta$, $\angle OCD=\varphi$.



Then $y=\sqrt{(c^2+x^2-2cx\cos\theta)}$, $c=\dfrac{a(m_1-m_2)}{2(m_1+m_2)}$.

$c=y\cos\varphi+x\cos\theta$. $\therefore \cos\varphi=\dfrac{c-x\cos\theta}{y}$.

Force acting on $D=(m_1+m_2)/y^2$.

Resolving this force, the component along OD

$$=(m_1+m_2)\cos(\theta+\varphi)/y_2=\frac{m_1+m_2}{y^3}\{(c-x\mathrm{sos}\theta)\cos\theta-\sin\theta\sqrt{[y^2-(c-x\cos\theta)^2]}\}$$

$$=\frac{(m_1+m_2)(c\cos\theta-x)}{(c^2+x^2-2cx\cos\theta)^{\frac{3}{2}}}.$$

$$\text{Work}=(m_1+m_2)\int_0^r\frac{(c\cos\theta-x)dx}{(c^2+x^2-2cx\cos\theta)^{\frac{3}{2}}}$$

$$=(m_1+m_2)\left(\frac{1}{\sqrt{(c^2+r^2-2cr\cos\theta)}}-\frac{1}{c}\right)$$

$$=2(m_1+m_2)^2\left(\frac{1}{\sqrt{4r^2(m_1+m_2)^2+a^2(m_1-m_2)^2-4ar(m_1^2-m_2^2)\cos\theta}}\right.$$

$$\left.-\frac{1}{m_1-m_2}\right).$$